CS744: Cyber-Physical Systems

Compositional Schedulability Analysis for Real-Time Systems

Insik Shin
Traditional Scheduling Framework

- Single real-time task in a single application
Hierarchical Scheduling Framework (HFS)

- Multiple tasks with a scheduler in a single application
Desired Properties for HSF

- Independent schedulability analysis
Desired Properties for HSF

- Clean separation between OS and application schedulers
Two-level Hierarchical Framework

- Deng & Liu’s approach [RTSS, 1997]
  - Uses EDF scheduler for OS scheduler, while OS scheduler should know task-level deadlines
  
    - Achieves independent schedulability analysis
    - No clean separation between schedulers
    - Limited to EDF scheduler for OS scheduler
Two-level Hierarchical Framework

- Kuo & Li’s approach [RTSS, 1999]
  - Uses **RM** scheduler for OS scheduler, while all applications have harmonic periods (multiple to each other)
  - Achieves independent schedulability analysis
  - Clean separation between schedulers, but the period constraints of applications due to OS scheduler
  - Limited to RM scheduler for OS scheduler
Compositional Scheduling Framework

- Insik Shin & Insup Lee,
  “Periodic Resource Model for Compositional Real-Time Guarantees”,
  RTSS ’03
Hierarchical Scheduling

- Open systems
  - Applications may be developed and validated independently in different environments

- Partitioning
  - Supporting temporal partitioning among applications for fault containment
Hierarchical Scheduling - Issues

- System-level scheduler's viewpoint

What is the real-time requirements of each application?
Hierarchical Scheduling - Issues

- Application-level scheduler’s viewpoint
Proposed Framework - Overview

- Interface-based hierarchical scheduling framework
Proposed Framework - Approaches

- Interface-based hierarchical scheduling framework

- Approach
  - Propose a new real-time resource model (periodic)
  - Extend real-time scheduling theories with the new resource model
  - Develop interfaces with these results
  - Use interfaces for component-based schedulability analysis
Proposed Framework - Assumptions

- Tasks
  - periodic
  - independent
  - fully preemptable
  - synchronously released

- Uni-processor scheduling

- Scheduling algorithms: EDF / RM
Outline

- Background - Real-time systems
- Proposed Framework
  - Motivation
  - Real-Time Resource Model
  - Schedulability Analysis
  - Component Timing Abstraction
  - Summary
Real-Time Resource Modeling

- **Real-time virtual resource model**
  - Characterize the timing property of resource allocations provided to a single task (application/component)

- **Previous approaches**
  - rate-based resource model

- **Our approach**
  - temporally partitioned resource model
Resource Modeling

- Dedicated resource
  - available at all times at full capacity

- Rate-based shared resource
  - available at fractional capacity at all times

- Time-shared resource
  - available at full capacity at some times
Periodic Resource Model

- Periodic resource model $\Gamma(P,Q)$
  - a time-shared resource,
  - characterizes periodic resource allocations
  - period $P$ and allocation time $Q$
  - Resource utilization $U_{\Gamma} = Q/P$
  - Example, $P = 3, Q = 2$
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Schedulability Analysis
Traditional Schedulability Analysis

- Demand-based analysis with dedicated resource

resource demand during an interval of length $t$ $\leq t$
Schedulability Analysis

- Demand- and supply-based analysis with periodic (time-shared) resource

\[
\text{resource demand, during an interval of length } t \leq \text{ resource supply, during an interval of length } t
\]
Resource Demand Bound

- Resource demand bound function
  - $dbf(W,A,t)$: the maximum possible resource demand of a task set $W$ under algorithm $A$ during an interval of length $t$
Demand Bound Functions

- For a periodic task set $W = \{T_i(p_i, e_i)\}$,
  - $\text{dbf}(W, A, t)$ for EDF [Baruah et al., ’90]
    
    $$
    \text{dbf} (W, \text{EDF}, t) = \sum_{T_i \in W} \left[ \frac{t}{p_i} \right] \cdot e_i
    $$
  
  - $\text{dbf}(W, A, t, i)$ for RM [Lehoczky et al., ‘89]
    
    $$
    \text{dbf} (W, \text{RM}, t, i) = e_i + \sum_{T_k \in HP(T_i)} \left[ \frac{t}{p_k} \right] \cdot e_k
    $$
Resource Supply

- Resource supply bound function
  - \( \text{sbf}_\Gamma(t) \): the minimum resource supply by resource \( \Gamma \) over all intervals of length \( t \)

- Periodic resource \( \Gamma(3,2) \)
Periodic Resource Model

- Supply bound function $sbf_{\Gamma}(t)$

\[
sbf_{\Gamma}(t) = \begin{cases} 
  t - (k+1)(P - Q) & \text{if } t \in [(k+1)P - 2Q, (k+1)P - Q] \\
  (k-1)P & \text{otherwise}
\end{cases}
\]
Schedulability Condition - EDF

A periodic task set $W$ is schedulable under EDF over the worst-case resource supply of periodic resource model $\Gamma(P,Q)$ if and only if

\[ \forall t > 0 \quad \text{dbf}(W, \text{EDF}, t) \leq t \quad \text{[Baruah et al. '90]} \]

\[ \forall t > 0 \quad \text{dbf}(W, \text{EDF}, t) \leq \text{sbrf}(t) \quad \text{[Shin & Lee, '03]} \]
Schedulability Condition - RM

- A periodic task set $W$ is schedulable under RM over the worst-case resource supply of periodic resource model $\Gamma(P, Q)$ if and only if

$$\exists 0 < t \leq p_i \ \forall T_i \in W \ \ \text{dbf}(W, \text{RM}, t, i) \leq \text{sbfr}(t)$$

[Shin & Lee, ’03]
Schedulability Analysis

- Demand- and supply-based analysis

naturally extensible with other scheduler, task models, and/or resource models, as long as they can provide resource demand and supply bounds.
Utilization Bounds

- Utilization Bounds
  - Utilization-based sufficient schedulability condition
  - Suppose $UB_{RM}=0.7$. Then a task set is schedulable under RM if $U_W \leq 0.7$.
  - For example, a task set $\{T_1(10,3), T_2(25,5)\}$ is schedulable since $U_W = 0.5$.

- Utilization of a task set $W$, $U_W = \sum \frac{e_i}{p_i}$
Utilization Bounds

- Utilization-based sufficient schedulability conditions
- Useful for online admission tests
- Previously developed for dedicated resource
- We generalize previous results with a periodic resource, with a new perspective
  - introducing an argument of the smallest task period
  - $\text{UB}_{\text{EDF}} \rightarrow \text{UB}_{\Gamma,\text{EDF}}(P_{\text{min}})$
Utilization Bound - EDF

- For dedicated resource, $UB_{EDF} = 1$ [Liu & Layland, ’73]
- For periodic resource $\Upsilon(P,Q)$,

$$UB_{\Upsilon, EDF}(P_{min}) = \frac{k \times U_{\Upsilon}}{k + 2(1 - U_{\Upsilon})}$$

- $P_{min}$ is the smallest task period (deadline) in a task set.
- $k$ represents the relationship between resource period $P$ and the smallest task period $P_{min}$, $k \approx \frac{P_{min}}{P}$
Utilization Bound - EDF

- For dedicated resource, $UB_{EDF} = 1$ [Liu & Layland, '73]
- For periodic resource $\Upsilon(P,Q)$,
  \[
  UB_{\Upsilon, EDF}(P_{\text{min}}) = \frac{k \times U_{\Upsilon}}{k + 2(1 - U_{\Upsilon})}
  \]
- When $U_{\Upsilon} = 1$, i.e., the periodic resource is a dedicated resource,
  \[
  UB_{\Upsilon, EDF}(P_{\text{min}}) = 1
  \]
- $U_{\Upsilon} = Q/P$
EDF Utilization Bound - Intuition

Observation

- schedulable iff $\text{dbf}(W,\text{EDF},t) \leq \text{sbf}_\Gamma(t)$, for all $t > P_{\min}$
EDF Utilization Bound - Intuition

- **Observation**
  - schedulable iff $\text{dbf}(W,\text{EDF},t) \leq \text{sbf}_\Gamma(t)$, for all $t > P_{\text{min}}$
  - $\text{dbf}(W,\text{EDF},t) \leq U_W \cdot t$
EDF Utilization Bound - Intuition

Observation

- schedulable iff \( \text{dbf}(W,\text{EDF},t) \leq \text{sbf}_\Gamma(t) \), for all \( t > P_{\text{min}} \)
- \( \text{dbf}(W,\text{EDF},t) \leq U_W \cdot t \), \( U_\Gamma(t-2(P-Q)) \leq \text{sbf}_\Gamma(t) \)
EDF Utilization Bound - Intuition

- **Observation**
  - schedulable iff \( \text{dbf}(W, EDF, t) \leq \text{sbf}_\Gamma(t) \), for all \( t > P_{\text{min}} \)
  - \( \text{dbf}(W, EDF, t) \leq U_w \cdot t, \ U_\Gamma(t-2(P-Q)) \leq \text{sbf}_\Gamma(t) \)
  - schedulable if \( U_w \cdot t \leq U_\Gamma(t-2(P-Q)) \), for all \( t > P_{\text{min}} \)
EDF Utilization Bound - Intuition

- **Observation**
  - schedulable iff $\text{dbf}(W,\text{EDF},t) \leq \text{sbf}_{\Gamma}(t)$, for all $t > P_{\min}$
  - $\text{dbf}(W,\text{EDF},t) \leq U_W \cdot t$, $U_{\Gamma}(t-2(P-Q)) \leq \text{sbf}_{\Gamma}(t)$
  - schedulable if $U_W \cdot t \leq U_{\Gamma}(t-2(P-Q))$, for all $t > P_{\min}$
  - schedulable if $U_W \leq \frac{U_{\Gamma}(t-2(P-Q))}{t}$, for all $t > P_{\min}$
Utilization Bound - RM

- Previous result [Liu & Layland, '73]
  \[
  UB_{RM}(n) = n \left( 2^n - 1 \right)
  \]

- We extend this earlier result, where \( k \approx \frac{P_{min}}{P} \).
  \[
  UB_{\Gamma, RM}(n, P_{min}) = U_{\Gamma} \times n \left( \frac{2k + 2(1 - U_{\Gamma})}{k + 2(1 - U_{\Gamma})} \right)^{\frac{1}{n}} - 1
  \]
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Component Timing Abstraction

- Abstracting the collective real-time requirements of a component as a single real-time requirement (real-time component interface)
Finding a periodic resource model $\Gamma(P,Q)$ that guarantees the schedulability of a component.
Abstraction - Example

- In this example, a solution space of a periodic resource $\Gamma(P,Q)$ is

Solution Space under EDF

- $\Gamma(P,Q)$
- EDF
- Periodic (50,7)
- Periodic (70,9)
Abstraction - Example

- An approach to pick one out of solution space
  - Given a range of $P$, we can pick $\Gamma(P,Q)$ such that $U_\Gamma$ is minimized. (for example, $28 \leq P \leq 46$)

![Graph showing resource period vs. resource capacity with example $\Gamma(29, 9.86)$]
Abstraction

\( U_w = 0.27 \)

\( U_\Gamma = 0.34 \)
Abstraction Overhead

- Abstraction overhead ($O_\Gamma$) is $\frac{U_\Gamma}{U_W} - 1$

- $U_W = 0.27$
- $U_\Gamma = 0.34$

- $\frac{0.34}{0.27} - 1 = 0.26$
Abstraction Overhead Bound

- Abstraction overhead ($O_{\Gamma}$) is

  - $A = \text{EDF}$  \quad $O_{\Gamma, \text{EDF}} \leq \frac{2 \times (1 - U_w)}{k + 2 \times U_w}$

  - $A = \text{RM}$  \quad $O_{\Gamma, \text{RM}} \leq \frac{1}{\log \left( \frac{2k + 2(1 - U_w)}{k + 2(1 - U_w)} \right)} - 1$
Abstraction Overhead

- Simulation Results
  - with periodic tasks and periodic resource under EDF/RM
  - the number of tasks $n : 2, 4, 8, 16, 32, 64$
  - the workload utilization $U(W) : 0.2 \sim 0.7$
  - ratio between the resource period and minimum task period : represented by $k$
Abstraction Overhead

\[ k \approx \frac{P_{\text{min}}}{P}, \quad U_W = 0.4, \quad n = 8 \]
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Summary

- Compositional real-time scheduling framework [RTSS ’03, ’04]
  - Periodic resource model
  - Schedulability analysis with periodic resource
  - Periodic interfaces for component-based analysis
Extension

- Compositional real-time scheduling framework
  - Periodic resource model [RTSS ’03]
    - independent tasks (without any synchronization)
    - uni-processor scheduling in a single node

- Extension
  - Synchronization protocols
  - Multi-core scheduling
  - End-to-end delay in distributed systems